K17U 1042

Reg. No. : ....

Name : ....

II Semester B.C.A. Degree (C.B.C.S.S. – Reg./Supple./Improv.) Examination, May 2017
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT-BCA : Mathematics for B.C.A. II
(2014 Admn. Onwards)

Time : 3 Hours   Marks : 40

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give a basis for the vector space $\mathbb{R}^3$.

2. Find the spectrum of $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$.

3. State the Cayley Hamilton theorem.

4. What is the smallest integer n such that the complete graph K_n has atleast 500 edges ? **(1×4=4)**

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. They carry **2** marks **each**.

5. Find the whole area of the curve $xy^2 = a^2 (a - x)$ and the y-axis.

6. Find the perimeter of the cardioid $r = a(1 - \cos\theta)$.

7. Find a 3 × 3 matrix A such that $A \neq 0$ but $A^2 = 0$.

8. Show that the diagonal elements of a skew symmetric matrix are all zero.

P.T.O.

9. Find the inverse of $\begin{bmatrix} -0.25 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$.

10. Prove that the determinant of an orthogonal matrix has value + 1 or − 1.

11. Let G be a (p, q) graph all of whose points have degree k or k + 1. If G has t > 0 points of degree k, show that t = p (k + 1) − 2q.

12. Give two non-isomorphic graphs with degree sequence (3, 2, 2, 1, 1, 1).

13. Draw the graph whose adjacency matrix is given by $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$. **(2×7=14)**

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. They carry **3** marks **each**.

14. Find the area common to the circles $r = a\sqrt{2}$ and $r = 2a \cos \theta$.

15. Obtain the intrinsic equation of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, the fixed point being the origin.

16. Solve by Gauss Elimination method.

$2x - y + z = 7$

$3x + y - 5z = 13$

$x + y + z = 5$

17. Find an eigenbasis and diagonalize $\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$.

18. Using Cayley Hamilton theorem find A^3 if $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$.

19. If A is the adjacency matrix of a graph with $V = (v_1, v_2, ..., v_p\}$, prove that for any $n \geq 1$, the $(i, j)^{th}$ entry of A^n is the number of $v_i - v_j$ walks of length n is G. **(3×4=12)**

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. They carry **5** marks **each**.

20. Find by double integration the area of the region enclosed by curves $x^2 + y^2 = a^2$, $x + y = a$ in the first quadrant.

21. Show that the inverse of an $n \times n$ matrix A exists if and only if rank $A = n$.

22. Find the eigenvalues and eigenvectors of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

23. Show that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$. **(5×2=10)**